K.P. College Manliparj Madhepura
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Time 12:30 PM to 1:20 PM
U.G. Semester - I, MIL-I
Date - 30/x/25

Matrices

* Definition of Matrix :- A rectangular array of mn No (Real or Complex) or members of a field (called elements) Containing m rows and n columns is called a matrix of order m×n

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & & a_{2n} \\ \cdots & \cdots & \cdots & & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

and It is denoted by $[a_{ij}]$

Operations of Matrices or Types of Matrices

Matrices

(I) Square Matrix
(II) Rectangular Matrix
(III) Row Matrix
(IV) Column Matrix
(V) Diagonal Matrix
(VI) Scalar Matrix
(VII) Unit Matrix
Triangular Matrix
Null Matrix or Zero Matrix

(I) Square Matrix :- The Matrix $[a_{ij}]$ $(i=1 \text{ to } m,\ j=1 \text{ to } n)$ is called Square if $m=n$, i.e if the No of row in a Matrix is equal to the No of Columns in that Matrix

Ex - $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}$ is Square Matrix

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$ is not a Square Matrix

(III) Rectangular Matrix :- A $m \times n$ Matrix is called Rectangular Matrix of $m \neq n$ i.e the No of row is not equal to the No of column.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3 \text{ order}}$$

(III) Row Matrix :- If a Matrix has only one row, the Matrix is called a row Matrices

Ex- $\begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix}_{1\times 3 \text{ order}}$

or

$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3 \text{ order}}$

(IV) Column Matrix :- If a matrix has only one column, the Matrix is called a column matrix

$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1 \text{ order}}$ or $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1 \text{ order}}$

(V) Diagonal matrix :- A square Matrix is said to be a diagonal matrix.

Ex- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times 3 \text{ order}}$ is a diagonal Matrix

(VI) Scalar Matrix :- If all the elements of a diagonal Matrix equal the Matrix is said to be a scalar matrix.

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}_{3\times 3 \text{ order}}$$

(VIII) Unit Matrix :- A diagonal matrix is said to be a unit matrix if each of the diagonal element is equal to unity.

For Ex- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$ is a unit matrix

or

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$ order of unit matrix.

(VIII) Null (or zero) matrix :- If all the elements of a matrix are zero, the matrix is called a Null matrix

For Ex- $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times2}$ or

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times3}$ or

(IX) Triangular Matrices :- A square Matrix, in which all the elements below its principal diagonal are zero is called an upper triangular Matrix.

Ex- $\begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$ is an upper triangular matrix

Ex- $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix}_{3\times3}$ is a lower triangular Matrix

Operation of Matrix Algebra

(i) addition of matrices

(ii) Scalar Multiplication of matrix (by a scalar)

and (iii) Multiplication of Matrices.

(i) Addition of Matrices or sum of Matrices

The sum of two matrices of the same order

$A = [a_{ij}]$ and $B = [b_{ij}]$ or $C = [c_{ij}]$

$c_{ij} = a_{ij} + b_{ij}$

Ex- Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \end{bmatrix}$

Hence A and B are conformal for addition

$$\therefore A+B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & 2+2 & 3+1 \\ 4+6 & 5+5 & 6+4 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 4 \\ 10 & 10 & 10 \end{bmatrix}$$

$$A + 0 = A$$

$$\text{and } 0 + A = A$$

Multiplication of Matrix by a scalar

$A + A$ is denoted by $2A$

If $A = [a_{ij}]$ then $A + A = [2a_{ij}]$ thus $2A = [2a_{ij}]$

Ex- If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Then $10A = \begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \end{bmatrix} = A \cdot 10$

$(k[a_{ij}] = k^n[a_{ij}])$

30/x/25