K.P. College Murliganj Madhepura

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Time 10:50 to 11:40 AM
Semester – I, MJC-01
Date – 30/X/25

Set Theory

Defn of set :-> No of well defined collection of object is called set.

Ex – $\{a, e, i, o, u\}$
or
$\{x / x \in$ set of vowels$\}$

Form of set

- Tabular form or Roster form or Listing form: $\{1, 2, 3 --- n\}$
- Set builder form or Rule form or property form: $\{x : x \in$ set of Natural no$\}$

Types of sets

- Singleton set
- pair set
- Finite set
- Infinite set
- Null set or Void set or Empty set or ϕ-set

(I) Singleton set :- The set having one and only one element is called a singleton set.

Ex – $\{0\}, \{1\}, \{2\}, \{x\}, \{y\}$ --- etc

(II) pair set :-> The set having two and only two element is called pair set.

Ex – $\{1,2\}$ $\{2,3\}$ $\{4,8\}$, $\{x,y\}$ ... etc

(III) Finite set :-> The process of counting the elements of a set comes to an end, the set is called a finite

$\{1, 2, 3, --- n\}$

⟨IV⟩ Infinite set :- If the process of counting of element of sets does not come to an end, the set is called an infinite set.

$$Ex - \{1, 2, 3, 4 \ldots\ldots \infty\}$$

(V) Null set or void set or ϕ sets :- A set having No element is called a Null set.

$$Ex - \{\}, \phi \ldots etc.$$

* $N \subset Z \subset Q \subset R \subset C$

Natural No $\subset$ integer $\subset$ Rational No $\subset$ Real No $\subset$ Complex No

Defn of Equality of set :- $A \subseteq B$ or $B \subseteq A$

Then $A = B$

$$Ex - A = \{1, 2, 3\} \quad B = \{3, 2, 1\}$$

$$\therefore A = B$$

Defn of Cartesian product :- The cartesian product $A \times B$, of the set A and B is the set of all possible ordered pair (a, b) where $a \in A$, $b \in B$.

i.e $A \times B = \{(a, b) / a \in A \text{ and } b \in B\}$

Ex - $A = \{1, 2, 3\}$. $B = \{4, 6\}$

Then $A \times B = \{(1,4)(1,6)(2,4)(2,6)(3,4)(3,6)\}$

$$* \quad \boxed{A \times B \neq B \times A}$$

Defn of Cartesian product of an indexed family of sets :- Let $A_1, A_2 -$ be sets Then The cartesian product is the set

$$A_1 \times A_2 \times A_3 \times A_4 \ldots \times A_n = \{(x_1, x_2, x_3, x_4 \ldots x_n) / x_i \in A_i \; \forall \; i = 1,2,3,4\}$$

$$R \times R \times R = \{(a_1, a_2, a_3)/ a_i \in R \; \forall \; i = 1,2,3\}$$

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Q:- proved that ⓐ $A\times(B\cup C) = (A\times B)\cup(A\times C)$

proof:- (a) :, let (x,y) be any elements of $A\times(B\cup C)$

$\Rightarrow (x,y) \in A\times(B\cup C)$

$\Rightarrow x \in A$ and $y \in (B\cup C)$

$\Rightarrow x \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow x \in A$ and $y \in B$ or $x \in A$ and $y \in C$

$\Rightarrow (x,y) \in A\times B$ or $(x,y) \in A\times C$

$\Rightarrow (x,y) \in (A\times B)\cup(A\times C)$

$\because A\times(B\cup C) \subseteq (A\times B)\cup(A\times C)$ —— ①

Conversely :. Let (x,y) be any element of $A\times B$

$\Rightarrow (x,y) \in (A\times B)\cup(A\times C)$

$\Rightarrow (x,y) \in A\times B$ or $x,y \in (A\times C)$

$\Rightarrow x \in A$ and $y \in B$ or $x \in A$ and $y \in C$

$\Rightarrow x \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow x \in A$ and $y \in (B\cup C)$

$\Rightarrow (x,y) \in A\times(B\cup C)$

$\therefore (A\times B)\cup(A\times C) \subseteq A\times B\cup C$ —— ②

From (1) and ② we get

$\therefore$ $\because A\times(B\cup C) = (A\times B)\cup(A\times C)$ proved.

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Matrices

Semester – 2, MJC – 01
Time – 11:45 AM to 12:30
Date – 30/x/25

Negative of a Matrix :– The negative of the Matrix $A = [a_{ij}]$ is defined to be the matrix B such that $A + B = 0$ (Null matrix)

Thus $B = [-a_{ij}] = (-1)[a_{ij}]$ i.e $(-1)A$

Ex – If $A = \begin{bmatrix} 3 & 7 & -5 \\ 2 & 3 & 9 \end{bmatrix}_{2\times 3 \text{ ord}}$

Then $-A = \begin{bmatrix} -3 & -7 & 5 \\ -2 & -3 & -9 \end{bmatrix}_{2\times 3 \text{ ord}}$

Difference of Matrix :– ~~Difference~~ $A - B$ between two Conformable matrices A and B is defined to be the Matrix $A + (-B)$

Ex – $A = \begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Hence $A-B$ exists

$\therefore A - B = \begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$= \begin{bmatrix} 4-1 & 5-2 & 6-3 \\ 7-4 & 8-5 & 9-6 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$

Commutativity of Matrix Addition :–

Question :– If the matrices A and B are Conformal for addition to prove that $A + B = B + A$

proof :– Let $A = [a_{ij}]$, $B = [b_{ij}]$ be two $m \times n$ matrices.

Then $A + B = [a_{ij}] + [b_{ij}]$

$= [a_{ij} + b_{ij}]$

But we know that the addition of number (or members of the field) is Commutative

$\therefore a_{ij} + b_{ij} = b_{ij} + a_{ij}$

$\therefore A + B = [b_{ij} + a_{ij}]$

$= [b_{ij}] + [a_{ij}] = B + A$

Hence the Commutative Law for addition of matrices holds.

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Associativity of Matrix addition :-

Question :- If A, B and C be three matrices conformable for addition to prove that $(A+B)+C = A+(B+C)$

Proof :- Let $A=[a_{ij}]$, $B=[b_{ij}]$ and $C=[c_{ij}]$ be matrices of the same order $m\times n$

Then $(A+B)+C = ([a_{ij}]+[b_{ij}])+[c_{ij}]$

$= ~~[a_{ij}+b_{ij}]+[c_{ij}]~~$

$= [(a_{ij}+b_{ij})+c_{ij}]$

$= [a_{ij}+b_{ij}+c_{ij}]$

$= [a_{ij}+(b_{ij}+c_{ij})]$

$= [a_{ij}]+[b_{ij}+c_{ij}]$

$= [a_{ij}]+([b_{ij}]+[c_{ij}])$

$= A+(B+C)$

Hence $(A+B)+C = A+(B+C)$

Thus the associative law for addition of Matrices holds

Question :- If $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix}$ and $B = \begin{pmatrix} 17 & -18 & 19 & -20 \\ 21 & -22 & 23 & -24 \\ -25 & 26 & -27 & 28 \end{pmatrix}$ find $A+B$ and $B+A$ and verify the commutative law of addition for these matrices.

Sol^n :-

$$A+B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix} + \begin{pmatrix} 17 & -18 & 19 & -20 \\ 21 & -22 & 23 & -24 \\ -25 & 26 & -27 & 28 \end{pmatrix}$$

$$= \begin{bmatrix} 1+17 & 2-18 & 3+19 & 4-20 \\ 5+21 & 6-22 & 7-23 & 8-24 \\ 9-25 & 10+26 & 11-27 & 12+28 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & -16 & 22 & -16 \\ 26 & -16 & 30 & -16 \\ -16 & 36 & -16 & 30 \end{bmatrix} \quad \text{———} (1)$$

Again $B+A = \begin{pmatrix} 17 & -18 & 19 & -20 \\ 21 & -22 & 23 & -24 \\ -25 & 26 & -27 & 28 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix}$

$$= \begin{pmatrix} 17+1 & -18+2 & 19+3 & -20+4 \\ 21+5 & -22+6 & 23+7 & -24+8 \\ -25+9 & 26+10 & -27+11 & 28+12 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & -16 & 22 & -16 \\ 26 & -16 & 30 & -16 \\ -16 & 36 & -16 & 30 \end{pmatrix} \text{ --- } (2)$$

From (1) and (2) we conclude that

$$A+B = B+A$$

Hence the Commutative Law of addition is verified.

Home work :- (1) Question :- If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $B = \begin{bmatrix} -10 & 11 & -12 \\ 13 & -14 & 15 \\ -16 & 17 & -18 \end{bmatrix}$

and $C = \begin{bmatrix} 0 & -2 & -3 \\ -4 & 0 & -6 \\ -7 & -8 & 0 \end{bmatrix}$ find $(A+B)+C$ and

$A+(B+C)$ and verify the associative Law of addition for these matrices.

(2) Question :- (i) Evaluate : $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 7 \\ 0 & -2 & 3 \end{bmatrix}$

(ii) Simplify : $2\begin{pmatrix} 3 \\ 2 \end{pmatrix} + 3\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

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