

Matrices

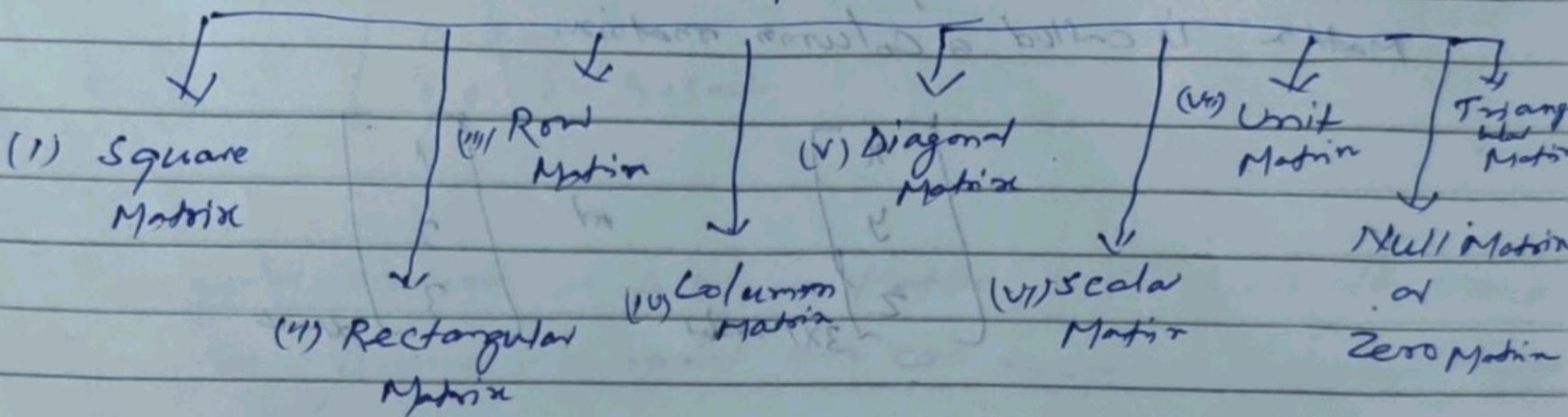
\* Definition of Matrix :- A rectangular array of  $m \times n$  (Real or Complex) or members of a field (called elements) containing  $m$  rows and  $n$  columns is called a matrix of order  $m \times n$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

and it is denoted by  $[a_{ij}]$

operations of Matrices or Types of Matrices

Matrices



(i) Square Matrix :-> The Matrix  $[a_{ij}]$  ( $i=1$  to  $m, j=1$  to  $n$ ) is called square if  $m=n$ , i.e. if the no. of rows in a Matrix is equal to the no. of columns in that Matrix

For Ex -  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3 \times 3}$  is Square Matrix

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3}$  is not a Square Matrix

(II) Rectangular Matrix: A  $m \times n$  Matrix is called Rectangular Matrix if  $m \neq n$  i.e. the No. of rows is not equal to the No. of columns.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3 \text{ ord}}$$

(III) Row Matrix: If a Matrix has only one row, the Matrix is called a row Matrix.

EX -  $\begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix}_{1 \times 3 \text{ ord}}$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1 \times 3 \text{ ord}}$$

(IV) Column Matrix: If a matrix has only one column, the Matrix is called a column matrix.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3 \times 1 \text{ ord}} \quad \text{or} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3 \times 1 \text{ ord}}$$

(V) Diagonal matrix: A square Matrix is said to be a diagonal matrix.

EX -  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  is a diagonal Matrix  
 $3 \times 3 \text{ ord}$

(VI) Scalar Matrix: If all the elements of a diagonal Matrix are equal the Matrix is said to be a scalar matrix.

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}_{3 \times 3 \text{ ord}}$$

(vii) Unit Matrix :- A diagonal matrix is said to be a unit matrix if each of the diagonal element is equal to unity.

For EX - 
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 is a unit matrix  $3 \times 3$

or 
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
 is a unit matrix  $2 \times 2$

(viii) Null (or zero) matrix :- If all the elements of a matrix are zero, the matrix is called a Null matrix

For EX - 
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
  $2 \times 2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
  $3 \times 3$

(ix) Triangular Matrices :- A square matrix, in which all the elements below its principal diagonal are zero is called an upper triangular matrix.

EX - 
$$\begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$
 is an upper triangular matrix  $3 \times 3$

EX - 
$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix}$$
 is a lower triangular matrix  $3 \times 3$

## Operations of Matrix Algebra

- (i) addition of matrices
- (ii) Scalar Multiplication of matrix (by a scalar)
- and (iii) Multiplication of Matrices

### (i) Addition of Matrices or sum of Matrices

The sum of two matrices of the same order

$$A = [a_{ij}] \quad \text{and} \quad B = [b_{ij}] \quad \text{or} \quad C = [c_{ij}]$$

$$C_{ij} = a_{ij} + b_{ij} \quad (\text{and } b_{ij} \text{ or } a_{ij})$$

Ex- let  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 3 & 1 \\ 6 & 5 & 4 \end{bmatrix}$

Hence A and B are conformable for addition

$$\therefore A+B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 1 \\ 6 & 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2+3 & 3+1 \\ 4+6 & 5+5 & 6+4 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 4 \\ 10 & 10 & 10 \end{bmatrix}$$

$$A+0 = A$$

$$\text{and } 0+A = A$$

### Multiplication of Matrix by a scalar

$A+A$  is denoted by  $2A$

$$\text{If } A = [a_{ij}] \text{ then } A+A = [2a_{ij}] \text{ thus } 2A = [2a_{ij}]$$

Ex- If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$$\text{The } 10A = \begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \end{bmatrix} = A \cdot 10$$

$$(kA)_{ij} = k \cdot a_{ij}$$

Solve  
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